GCE Examinations

Further Pure Mathematics Module FP1

Advanced Subsidiary / Advanced Level

Paper A

Time: 1 hour 30 minutes

Instructions and Information

Candidates may use any calculator except those with a facility for symbolic algebra and/or calculus.

Full marks may be obtained for answers to ALL questions.

Mathematical and statistical formulae and tables are available.

This paper has 8 questions.

Advice to Candidates

You must show sufficient working to make your methods clear to an examiner. Answers without working will gain no credit.



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1.
$$f(z) \equiv z^3 - 5z^2 + 17z - 13$$
.

- *(a)* Show that (z - 1) is a factor of f(z).
- Hence find all the roots of the equation f(z) = 0, giving your answers in the form a + ib*(b)* where *a* and *b* are integers.

(1 mark)

(5 marks)

(6 marks)

(2 marks)

2. Find the general solution of the differential equation

$$x\frac{\mathrm{d}y}{\mathrm{d}x} + 3y = \frac{\mathrm{e}^x}{x^2},$$

giving your answer in the form y = f(x).

Express $\frac{1}{r(r+1)}$ in partial fractions. *(b)* Hence, or otherwise, find

$$\sum_{r=3}^{35} \ \frac{1}{r(r+1)},$$

giving your answer as a fraction in its lowest terms. (4 marks)

4. Find the set of values of *x* for which

$$\frac{(x-3)^2}{x+1} < 2.$$
 (7 marks)

5. Sketch the curve with polar equation $r = a \cos 3\theta$, a > 0, for $0 \le \theta \le \pi$. (3 marks) *(a)*

Show that the total area enclosed by the curve $r = a \cos 3\theta$ is $\frac{\pi a^2}{4}$. (6 marks) *(b)*

3.

(a)



Fig. 1

Figure 1 shows the curves $y = 2 \cos x$ and $y = e^x$ in the interval $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$.

Given that $f(x) \equiv e^x - 2 \cos x$,

(a) write down the number of solutions of the equation f(x) = 0 in the interval $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$.

(1 mark)

(b) Show that the equation f(x) = 0 has a solution, α , in the interval [0, 1]. (2 marks)

(c) Using 0.5 as a first approximation to α , use the Newton-Raphson process once to find an improved estimate for α , giving your answer correct to 2 decimal places. (4 marks)

(d) Show that the estimate of α obtained in part (c) is accurate to 2 decimal places. (2 marks)

There is another root, β , of the equation f(x) = 0 in the interval [-2, -1].

(e) Use linear interpolation once on this interval to estimate the value of β , giving your answer correct to 2 decimal places.

(3 marks)

Turn over

7. The complex numbers *z* and *w* are such that

$$z = \frac{A}{1-i}$$
 and $w = \frac{B}{2+i}$,

where *A* and *B* are real.

Given that z + w = 6,

z and w are represented by the points P and Q respectively on an Argand diagram.

(b)	Show P and Q on the same Argand diagram.	(5 marks)
(c)	Find the distance PQ in the form $a\sqrt{5}$.	(3 marks)

8. (a) Find the values of p and q such that $x = p \cos t + q \sin t$ satisfies the differential equation

$$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} + 4\frac{\mathrm{d}x}{\mathrm{d}t} + 3x = \sin t.$$
 (6 marks)

(b) Hence find the solution of this differential equation for which x = 1 and $\frac{dx}{dt} = \frac{1}{2}$ at t = 0.

(9 marks)

END